Exam. Code : 103205

Subject Code : 1190

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—II

(Number Theory)

Time Allowed—3 Hours] [Maximum Marks—50

Note :—Attempt **FIVE** questions, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

I. (a) Prove by mathematical induction that $3^{4n+1} + 2^{2n+2}$ is divisible by 7 for every natural number n. 5

(b) Show that the product of any m consecutive integers is divisible by $\lfloor m$. 5

II. (a) If $c \mid a$, $c \mid b$ and $\gcd\left(\frac{a}{c}, \frac{b}{c}\right) = 1$, show that $\gcd(a, b) = c$. 5

(b) Find all the solutions in positive integers of the Diophantine equation $15x + 7y = 111$. 5

SECTION—B

III. (a) If x and y are positive real numbers, prove that $[x]\,[y] \leq [xy]$; $[x]$, $[y]$, $[xy]$ are greatest integer functions. 5

(b) Find the number and sum of the divisors of the natural number 10800. 5

IV. (a) For even integer n, prove that $\phi(2n) = 2\phi(n)$, ϕ is Euler's phi function. 5

(b) Verify Mobius Inversion formula for $n = 24$. 5

SECTION—C

V. (a) If $a \equiv b \pmod{m}$, then prove that $a^p \equiv b^p \pmod{m}$, $p \in N$. 5

(b) Solve $13x \equiv 3 \pmod{47}$. 5

VI. (a) Prove that the linear congruence $ax \equiv b \pmod{m}$ has a solution iff $d \mid b$ where $d = \gcd(a, m)$. 5

(b) Solve $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$. 5

SECTION—D

VII. (a) If $\gcd(a, 133) = \gcd(b, 133) = 1$, then using Fermat's theorem prove that $a^{18} \equiv b^{18} \pmod{133}$. 5

(b) If $a^p \equiv b^p \pmod{p}$ for any prime p, then prove that $a \equiv b \pmod{p}$. 5

VIII. (a) Show that $\lfloor 18 \equiv -1 \pmod{437}$. 5

(b) Using numerical, encipher the word 'STUDY' and decipher the word 'ERRN'. 5